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SKEWNESS CORRECTED CONTROL CHARTS: A NEW PROBABILITY MODEL

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Abstract
In the industry, control charts are powerful statistical tools for process control. Shewhart control charts assume a Normal Distribution for the quality characteristic. On the basis of sample size n, if 't_n' is a statistic, Shewhart variable control charts have control limits $E(t_n) \pm 3S.E(t_n)$. In the case of a skewed population, control limits must be constructed using a different method. A skewness correction method has been used to construct control charts for mean and range by several researchers.
By adopting the popular probability model Exponentiated Inverse Rayleigh Distribution (EIRD), this paper attempts to construct a control chart with skewness corrected, using EIRD. Using coefficients of skewness techniques Bowley's and Kelly's as a basis, we construct "variable control charts for the mean and range of subgroups in EIRD". Coverage probabilities are also calculated based on the simulation technique. The findings are compared with the methods of EIRD and other existing models.
Keyword: Exponentiated probability model, shewart control charts, skewness corrected (S.C) control charts, Bowley's and Kelly's method.

INTRODUCTION

The probability density function (pdf) of EIRD is

$$f(x) = \frac{2\beta\delta^2}{x^3} e^{-(\delta/x)^2}[1-e^{-(\delta/x)^2}]^{\beta-1}, x \geq 0, \delta > 0, \beta > 0 \quad (1.1)$$

Where δ - scale parameter and β - shape parameter

When β = 1 equation (1.1) reduces to Inverse Rayleigh Distribution.

The cumulative distribution function (cdf) is

$$F(x) = 1 - \left[1 - e^{-(\delta/x)^2}\right]^{\beta}$$

The Reliability function is given by

$$R(x) = 1 - F(x) = \left[1 - e^{-(\delta/x)^2}\right]^{\beta}$$

The Hazard function is

$$h(x) = \frac{f(x)}{R(x)} = 2\beta\delta^2 x^{-3} e^{-(\delta/x)^2}\left[1 - e^{-(\delta/x)^2}\right]^{-1}$$

The r[th] moment about origin is

$$\mu_r^{'} = \beta \sum_{j=0}^{\infty} \binom{\beta - 1_c}{j} (-1)^j \frac{\left[\delta^2 (j+1)\right]^{r/2}}{j+1} \Gamma\left(1 - \frac{r}{2}\right)$$

Bowley's Coefficient of Skewness = $\frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1}$ = 0.46089

Kelly's Coefficient of Skewness = $\frac{P_{10} + P_{90} - 2P_{50}}{P_{90} - P_{10}} = 0.76335$

There are several authors who have studied control charts for many different types of probability distributions, including the mean and range of symmetric and skewed distributions. Chan & Cui (2003) [1], Kantam et al. (2006) [2], Subbarao & Kantam (2008) [3], Chaitanya Priya (2011) [4], Srinivasa Rao & Kantam (2012) [5], Srinivasa Rao and Srinivasa Kumar (2015) [6], Sriram et al. (2016) [7], Subbarao et al. (2016) [8].

Our efforts in this paper were motivated by these studies in which δ =1 and β = 0.5 were used to evaluate control charts for process variates which follows EIRD. Section II presents a brief summary of Chan & Cui (2003). In Section III, Bowley's and Kelly's coefficients of skewness are used to determine control chart constants for EIRD. EIRD coverage probabilities are discussed in Section IV using the Bowley and Kelly approach. The EIRD control charts are compared in the V section to the Inverse Rayleigh Distribution (IRD) and Inverse Half Logistic Distribution (IHLD) created by Subbarao et al. (2016) using skewness-corrected control charts. Section VI provides the conclusions on the two approaches and the probability models.

PRINCIPLE OF SKEWNESS CORRECTED CONTROL CHART (SUMMARY OF CHAN AND CUI 2003)

A process variate X is considered to have a normal distribution with mean μ standard deviation σ. In this quality variate, let $x_1, x_2, \ldots, x_n$ represent a subgroup of 'n' measurements of the quality characteristic. According to Shewhart, the statistical quality control limits for the mean and standard deviation of a process are as follows:

Shewhart $\overline{\overline{X}}$ Chart:

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R},\ CL_{\overline{X}} = \overline{\overline{X}},\ LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

Shewhart R Chart:

$$UCL_R = D_4\overline{R}, CL_R = \overline{R}, LCL_R = D_3\overline{R}$$

where: $\overline{\overline{X}}$ Grand Average, $\overline{R}$: Average Range. In any standard textbook on statistical quality control, there are available constants A_2, D_3, D_4 for specific sub-group sizes. There must be a non-zero coefficient of skewness if the process quality variable does not follow a normal distribution, which is known by its mathematical structure or can be estimated from a sample data set with the use of empirical methods. We will denote it by k_3. Skewness Corrected (SC) control charts for $\overline{X}$ charts have the following control limits and central line:

SC $\overline{X}$ Chart:

$$\begin{cases} UCL_{\overline{X}} = \overline{\overline{X}} + \left(3 + \dfrac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\dfrac{\overline{R}}{d_2^*\sqrt{n}} \equiv \overline{\overline{X}} + A_U^*\overline{R} \\ CL_{\overline{X}} = \overline{\overline{X}} \\ LCL_{\overline{X}} = \overline{\overline{X}} + \left(-3 + \dfrac{4k_3/(3\sqrt{n})}{1+0.2k_3^2/n}\right)\dfrac{\overline{R}}{d_2^*\sqrt{n}} \equiv \overline{\overline{X}} - A_L^*\overline{R} \end{cases} \quad (2.1)$$

The constant d_2^* was created and reported in Chan and Cui (2003). The findings of the SC technique control limits for n = 2 (1) 5, 7, and 10 are tabulated by Chan and Cui (2003). It is advised that we choose the closest value of k_3 or utilize interpolation when the value of k_3 for our chosen model does not appear in Table 1 of Chan and Cui (2003).

As previously, the control limits for the range chart with skewness correction are provided

SC R Chart:

$$\begin{cases} UCL_R = \left[1 + (3 + d_4^*)\dfrac{d_3^*}{d_2^*}\right]\overline{R} \equiv D_4^*\overline{R} \\ CL_R = \overline{R} \\ LCL_R = \left[1 + (-3 + d_4^*)\dfrac{d_3^*}{d_2^*}\right]\overline{R} \equiv D_3^*\overline{R} \end{cases} \quad (2.2)$$

where the control chart constants were d_2^*, d_3^*, d_4^* created specifically to account for the model's non normality. Table 2 of Chan and Cui (2003) gives a SC constants for Range chart.

If the distribution under examination is skewed, any common formula is used to get the coefficient of skewness, say k_3. If necessary, linear interpolation is used to determine the control limits A_L^*, A_U^* from the bivariate Table 1of Chan and Cui (2003), specifically for the subgroup size when a control chart for mean is required.

The pair (A_L^*, A_U^*) so chosen would provide the control limits of the $\overline{X}$ chart based on SC technique utilized in the equation (2.1). A similar process might be used for range charts based on SC as well.

CONTROL CHART CONSTANTS FOR EXPONENTIAL INVERSE RAYLEIGH DISTRIBUTION

Section I lists the basic characteristics of EIRD. Bowley's and Kelly's methods are used to determine the coefficient of skewness as EIRD is skewed distribution. The formulas are given below:

$$k_{3(B)} = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{Q_3 - Q_1}$$

$$k_{3(k)} = \frac{P_{90} - 2P_{50} + P_{10}}{P_{90} - P_{10}}$$

$Q_i (i = 1,2,3)$ and $P_i (i = 10,50,90)$ are i^th^ quartile and percentile of the EIRD.

For developing control chart constants, we set the EIRD parameters to δ=1, β =0.5. For EIRD, the skewness coefficients for Bowley and Kelly are 0.46089 and 0.76335. As can be observed from Table 1 of Chan and Cui (2003) does not include the values of our coefficients of skewness. To obtain the values $A_L^*, A_U^*, D_3^*, D_4^*$ of that correspond to the k_3 values under consideration, we have therefore turned to linear interpolation.

According to the linear interpolation technique, the values of $A_L^*, A_U^*, D_3^*, D_4^*$ are as follows and are given in Table 3.1 and Table 3.2 for the chosen values of n and k_3. As a result, using any one of these approaches may not cause the Upper Control Limit and Lower Control Limit numbers to alter.

For developing control chart constants, we set the EIRD parameters to δ=1, β =0.5. For EIRD, the skewness coefficients for Bowley and Kelly are 0.46089 and 0.76335. As can be observed from Table 1 of Chan and Cui (2003) does not include the values of our coefficients of skewness. To obtain the values $A_L^*, A_U^*, D_3^*, D_4^*$ of that correspond to the k_3 values under consideration, we have therefore turned to linear interpolation.

Table 3.1. S.C. $\overline{X}$- Chart Constants using Interpolation

	Coefficient of Skewness for Bowley = 0.46089		Coefficient of Skewness for Kelly= 0.76335	
n	A_U^*	A_L^*	A_U^*	A_L^*
2	2.17502	1.63955	2.34907	1.48820
3	1.14827	0.90782	1.23908	0.84728
4	0.82761	0.67782	0.86545	0.61728
5	0.63761	0.52543	0.67545	0.50273
7	0.45456	0.38696	0.47727	0.37182
10	0.33305	0.28848	0.34818	0.28091

Table 3.2. S.C. R - Chart Constants using Interpolation

	Coefficient of Skewness for Bowley = 0.46089		Coefficient of Skewness for Kelly = 0.76335	
n	D_4^*	D_3^*	D_4^*	D_3^*
2	4.24045	0	4.39180	0
3	3.09349	0	3.25998	0
4	5.25761	0.01365	2.83544	0.06454
5	2.43197	0.14457	2.59089	0.16727
7	2.19045	0.27305	2.34180	0.28818
10	2.00893	0.38152	2.15271	0.38909

When δ=1, β =0.5 EIRD generates random samples of size 5 (5) 25. The mean and range values are computed for each sample. The overall average and the average of the ranges were also determined. The upper and lower control limits of the $\overline{X}$ -chart and the R-chart for the EIRD are derived using the constants A_L^* , A_U^* and D_3^*, D_4^* are given in Tables 3.3 and 3.4.

Table 3.3. S.C. Control Limits for $\overline{X}$ - Chart

	Coefficient of Skewness for Bowley		Coefficient of Skewness for Kelly	
n	**LCL**	**UCL**	**LCL**	**UCL**
2	0	43.79437	0	46.43200
3	0	51.25566	0	54.24846
4	0	45.62498	0	47.14390
5	0	46.13751	0	48.12511
7	0	44.55489	0	46.15135
10	0	87.26291	0	90.21569

Table 3.4. S.C Control Limits for R - Chart

	Coefficient of Skewness for Bowley		Coefficient of Skewness for Kelly	
n	**LCL**	**UCL**	**LCL**	**UCL**
2	0	64.26335	0	66.55704
3	0	101.95134	0	107.43831
4	0.54511	209.96089	2.57738	113.23233
5	0.75938	127.74279	8.78610	136.09031
7	19.19478	153.98353	20.25838	164.62308
10	74.45748	392.06299	75.93485	420.12312

CALCULATION OF COVERAGE PROBABILITIES

The amount of sub-group averages and ranges that fall inside the Lower and Upper control limits are counted out of 10,000 simulation runs. The proportion of sample values that are inside the control limits will be determined. These ratios for the $\overline{X}$ and R-Charts are referred to as the coverage probability of the relevant pair of control limits. For EIRD model we computed the equivalent skewness-adjusted coverage probabilities for mean and range charts which are shown in Tables 4.1 and 4.2 for both Bowley's and Kelly's techniques.

Table 4.1. Coverage Probabilities for $\overline{X}$- Chart

n	EIRD	
	Bowley's	**Kelly's**
2	0.9725	0.9742
3	0.9782	0.9795
4	0.9750	0.9765
5	0.9714	0.9724
7	0.9719	0.9731
10	0.9863	0.9865

Table 4.2. Coverage Probabilities for R- Chart

n	EIRD	
	Bowley's	**Kelly's**
2	0.9762	0.9769
3	0.9723	0.9735
4	0.8353	0.4295
5	0.4238	0.3782
7	0.2428	0.2349
10	0.0988	0.0977

Because the statistical theory underlying the coverage probabilities is the well-known inclusion probability of 0.9973 within them, they will provide a clue as to how reliable the control limits are. It is not necessary for our evolved control limits to precisely cover 0.9973 probabilities because we used empirical skewness coefficients such as those suggested by Bowley and Kelly. Being the widely used confidence coefficient in statistical inference, any coverage probability above 0.95 may be considered acceptable, and the related control limits may be used with comfort. Because these probabilities are based on empirical measurements, no consistent upward or downward trend can be anticipated. The main benefit of using these coverage probabilities is that a user can reduce the risk of the conclusions by adjusting the subgroup size and empirical coefficient of skewness by looking for suitable coverage probabilities over 0.95.

COMPARISON OF EIRD WITH IRD AND IHLD

The control limits and related coverage probabilities for the probability models EIRD, IRD, and IHLD with respect to both approaches are shown separately for $\overline{X}$ and R charts in Tables 5.1, 5.2, 5.3, and 5.4 respectively, for simple comparison and quick reference. Table 5.5 provides the corresponding skewness adjusted coverage probabilities for mean and range charts using EIRD, IRD, and IHLD for both Bowley's and Kelly's techniques.

Table 5.1. Consolidated Table of Bowley's Coefficient of Skewness Corrected Control Limits for $\overline{X}$- Chart

n	EIRD			IRD			IHLD		
	Lower CL	**Upper CL**	**Coverage Probabilities**	**Lower CL**	**Upper CL**	**Coverage Probabilities**	**Lower CL**	**Upper CL**	**Coverage Probabilities**
2	0	43.79437	0.97250	0.52016	3.23611	0.88520	0	21.84128	0.97240
3	0	51.25566	0.97820	0.17616	3.63997	0.93850	0	25.57480	0.97810
4	0	45.62498	0.97500	0.28230	3.47268	0.93100	0	22.78219	0.97490
5	0	46.13751	0.97140	0.01611	3.79623	0.96110	0	23.03672	0.97130
7	0	44.55489	0.97190	0.22889	3.52110	0.95920	0	22.24492	0.97170
10	0	87.26291	0.98630	0.27687	3.45636	0.96930	0	43.56914	0.98630

Table 5.2. Consolidated Table of Kelly's Coefficient of Skewness Corrected Control Limits for $\overline{X}$- Chart

n	EIRD			IRD			IHLD		
	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities
2	0	46.43200	0.97420	0.60951	3.34201	0.84800	0	23.16348	0.97390
3	0	54.24846	0.97950	0.27085	3.75642	0.94260	0	27.07529	0.97940
4	0	47.14390	0.97650	0.36705	3.55763	0.93660	0	23.53778	0.97630
5	0	48.12511	0.97240	0.09106	4.51245	0.97570	0	24.03353	0.97230
7	0	46.15135	0.97310	0.28573	3.59331	0.96140	0	23.04372	0.97300
10	0	90.21569	0.98650	0.32046	3.51954	0.97170	0	45.05309	0.98640

Table 5.3. Consolidated Table of Bowley's Coefficient of Skewness Corrected Control Limits for R- Chart

n	EIRD			IRD			IHLD		
	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities
2	0	64.26335	0.97620	0	2.99538	0.95460	0	32.11705	0.97610
3	0	101.95134	0.97230	0	5.11345	0.93150	0	50.93818	0.97220
4	0.54511	209.96089	0.83530	0.01608	5.64832	0.94000	0.35181	54.17894	0.76730
5	0.75938	127.74279	0.42380	0.42592	7.74503	0.91730	3.78209	63.82471	0.42860
7	19.19478	153.98353	0.24280	1.03089	8.37537	0.75320	9.58915	76.90672	0.24380
10	74.45748	392.06299	0.09880	1.99282	10.03437	0.71390	37.22532	195.66479	0.09900

Table 5.4. Consolidated Table of Kelly's Coefficient of Skewness Corrected Control Limits for R - Chart

n	EIRD			IRD			IHLD		
	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities	Lower CL	Upper CL	Coverage Probabilities
2	0	66.55704	0.97690	0	3.06481	0.95560	0	33.26699	0.97670
3	0	107.43831	0.97350	0	5.30602	0.93580	0	53.69154	0.97330
4	2.57738	113.23233	0.42950	0.06984	5.85723	0.92400	1.26132	56.60364	0.43610
5	8.78610	136.09031	0.37820	0.49337	8.08133	0.91160	4.38175	68.01183	0.38110
7	20.25838	164.62308	0.23490	1.08813	8.76539	0.74020	10.12403	82.24146	0.23590
10	75.93485	420.12312	0.09770	1.96821	10.52462	0.70470	37.95752	209.72314	0.09780

Table 5.5. Coverage Probabilities

n	Mean Chart						Range Chart					
	EIRD		IRD		IHLD		EIRD		IRD		IHLD	
	Bowley's	Kelly's	Bowley's	Kelly's	Bowley's	Kelly's	Bowley's	Kelly's	Bowley's	Kelly's	Bowley's	Kelly's
2	0.97250	0.97420	0.88520	0.84800	0.97240	0.97390	0.97620	0.97690	0.95460	0.95560	0.97610	0.97670
3	0.97820	0.97950	0.93850	0.94260	0.97810	0.97940	0.97230	0.97350	0.93150	0.93580	0.97220	0.97330
4	0.97500	0.97650	0.93100	0.93660	0.97490	0.97630	0.83530	0.42950	0.94000	0.92400	0.76730	0.43610
5	0.97140	0.97240	0.96110	0.97570	0.97130	0.97230	0.42380	0.37820	0.91730	0.91160	0.42860	0.38110
7	0.97190	0.97310	0.95920	0.96140	0.97170	0.97300	0.24280	0.23490	0.75320	0.74020	0.24380	0.23590
10	0.98630	0.98650	0.96930	0.97170	0.98630	0.98640	0.09880	0.09770	0.71390	0.70470	0.09900	0.09780

CONCLUSIONS

The following observations are made after careful examination of the coverage probabilities listed in Table 5.5.

$\overline{X}$ - Chart:

- Kelly's technique is preferred when comparing Bowley's and Kelly's for EIRD, IRD, and IHLD.

- Of the three probability models for EIRD, IRD, and IHLD, EIRD has higher coverage probabilities than IRD and IHLD.

R- Chart:

- For EIRD, IRD, and IHLD, when n = 2 and n = 3, Kelly's method is preferred, whereas for the remaining cases, Bowley's method has higher coverage probability. As a result, we can conclude that Bowley's technique is better when n is large.
- In comparison to existing models, the coverage probabilities for the probability model EIRD when n = 2 and n = 3 are higher.

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